BUCKLING AND FREE VIBRATION ANALYSIS OF SANDWICH BEAM WITH MR CORE

A thesis submitted in the partial fulfilment

Of the requirements for the degree of

Master of Technology

in

Mechanical Engineering

(Specialization: Machine Design and Analysis)

Submitted by

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(ROLL NO. 213ME1398)

DEPARTMENT OF MECHANICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

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<u>CERTIFICATE</u>

This is to certify that the thesis entitled, "**BUCKLING AND FREE VIBRATION ANALYSIS OF SANDWICH BEAM WITH MR CORE**" submitted by Mr. **DESHMUKH AKSHAY VYANKATRAO (213ME1398)** in partial fulfilment of the requirements for the award of **Master Of Technology** degree in **Mechanical Engineering** with specialization in **Machine Design And Analysis** at the National Institute of Technology, Rourkela (India) is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other University / Institute for the award of any Degree or Diploma.

DATE

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ABSTRACT

The present study deals with the buckling and free vibration analysis of sandwich beam with Magneto rheological fluid as a core material. Beam with different boundary conditions such as Fixed-fixed, fixed-pinned, fixed-free and pinned-pinned has been investigated. The beam is modelled with the help of finite element method where each node of sandwich beam contain six degree of freedom. By using Hamilton principle governing equation of motion is derived with a help of FEM. A ten element discretization satisfy the convergence requirement and validation of formulation is done by comparing first three natural frequency with those available in literature. The effect of core thickness parameter, magnetic field strength and shear parameter on buckling load parameter, natural frequency parameter and modal loss factor have been studied.

Keywords: FEA, Sandwich beam, MR fluid, loss factor.

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NOMENCLATURE:

$\tau_{y(H)}$	: Shear stress due to magnetic field
μ_p	: Plastic viscosity
$\dot{\gamma}$	: Rate of shear strain
A	: Cross section area
E	: Young's Modulus
L	: Length of beam
$\{\Delta^e\}$	: Deformation matrix
G^*	: Shear modulus of MR fluid
G_r	: Shear modulus of sealing rubber
u1	: Longitudinal displacement of bottom layer
u3	: Longitudinal displacement of top layer
w1	: Transverse displacement of bottom layer
w3	: Transverse displacement of top layer
ϴ1	: Angular deformation of bottom layer
ϴ3	: Angular deformation of top layer
U(e)	: Elemental potential energy
Uk(e)	: Elemental potential energy of constraining layer
Uv(e)	: Elemental potential energy of viscoelastic core
$T_k^{(e)}$	: Elemental kinetic energy of constraining layer
$T_v^{(e)}$	: Elemental kinetic energy of viscoelastic core
$\overline{G}$	: Homogeneous shear modulus of MR fluid
$W_p^{(e)}$	: Work done by axial load
$K_g^{(e)}$	: Elemental geometric matrix
K	: stiffness matrix
M	: mass matrix

Chapter No. 1

INTRODUCTION

1.0 <u>INTRODUCTION</u>

Knowledge of damping characteristics requires to study of the dynamic performance of structures. The damping factor plays an important role in the estimation of structures and materials for their performance under noise and vibration control circumstances. Viscoelastic materials in suitable arrangement incorporating vibration control of machines and structures is an important aspect of investigation [1]. Vibration is main cause of impact on the engineering structures and their performance frequently, damping in structures stimuli its performance. To control an undesired effect due to vibration there are many damping mechanisms are developed over time. Whereas in some cases vibration is desirable such as tuning fork, mobile phone etc. Basically damping known as to the removal of mechanical energy from a vibrating system, mainly by changing the mechanical energy into heat energy by means of some dissipation mechanism. Commonly internal structural damping is provided by all materials. It is not substantially effective to minimize the vibration around resonant frequencies most of the time. Hence, by putting these materials in connection with the highly damped and dynamically stiffed material it is possible to control the vibration.

1.1 Sandwich beam:

Sandwich beam consist of three layered like two elastic layer separated by viscoelastic layer is used to reduce a vibration where viscoelastic material itself act as damping.

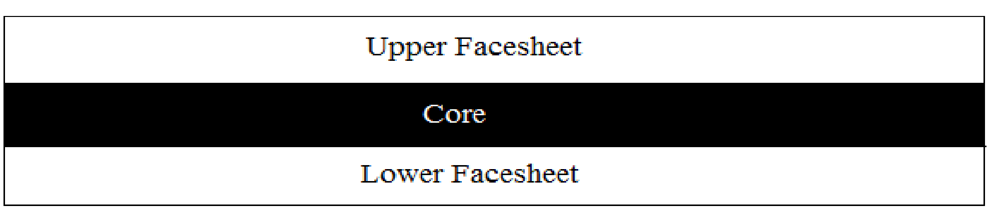

Figure 1 Sandwich Beam

Visco-elastic material exhibit properties of both elastic and viscous material. If elastic material is subjected to a load within elastic limit and then removed it will regain its equilibrium position within shortest time but in case of viscoelastic material it will take some time to gain its original position. There are two methods for surface treatment of viscoelastic material to improve its damping capacity i.e. constrained layer [2] and unconstrained layer or free layer treatment.

1.2 Why Sandwich Structure?

It is known that separating two material with lightweight material in between will increase structural stiffness and strength at very low weight and cost. This distinction along with many drivers such as environmental benefits, cost saving, freedom of design are making sandwich beam more popular in structural design.

1.2.1 Weight benefits:

Use of sandwich beam is because of its lightweight in construction. In transportation low weight enables to carry more payload. Sport equipment in which strength and speed is more important.

1.2.2 Design Benefits:

Sandwich structure is famous for its freedom in design .Conventional material such as wood and metal are restricted for its limit in their shape whereas sandwich structure can be shapeable in any kind of form until the final stage of production. This allows for non-linear and smooth design which is not only for esthetic reason but also for aerodynamic reason.

1.2.3 Additional benefits:

Thermal insulation.

Non corrosive.

Sound insulation.

Very low water absorption.

1.3 Structural Requirements:

Strength:

Viscoelastic cores and some face materials are guiding with regards to mechanical properties and care must be taken to confirm that the materials are positioned in the panel to take the best benefit of this attribute.

Stiffness:

At very low weights sandwich structures are frequently used to increase the stiffness. Because most of the core materials have relatively low shear modulus, however, for shear deflection of the structure the deflection calculations must allow in addition to the bending deflections consideration.

Adhesive Performance:

In order to loads to be transmitted from one facing to the other the adhesive must strictly attach the facings to the core material. Appropriate adhesives include high strength materials available as liquids, high modulus, pastes or dry films. As a general rule, relatively brittle adhesive or a low peel-strength should never be used with very light sandwich structures which may be exposed to damage in storage, handling or service.

Economic Considerations:

A cost effective solution provide composite sandwich panels. Value analysis ought to include assessment of production, installation costs and assembly costs including supporting structure.

1.4 How Sandwich Structure Works:

1.4.1 Loads:

Consider a cantilever beam with free end subjected to a load. The fixed end subjected to maximum bending moment due to applied load and a shear force along the length of the beam.

These forces create tension in the upper covering and compression in the lower covering in a sandwich panel. To make the composite panel work as a homogeneous structure the core spaces the facing coverings and transfers shear between them.

1.4.2 Deflections:

The deflection of a sandwich panel is cause due to bending and shear components. The shear deflection is dependent on the shear modulus of the core. The bending deflection is dependent on the relative tensile and compressive moduli of the covering materials.

Total Deflection = Bending Deflection + Shear Deflection.

1.5 Failure Mode:

All potential failure modes are considered in their analysis of sandwich panels by designers. A summary of the key failure modes is shown below:

Strength

The tensile, compressive and shear stresses induced by the applied load should be able to take by the covering and core materials. The covering to core adhesive must be capable of conveying the shear stresses between cover and core.

Panel buckling

To prevent the panel from buckling under end compression loads the core thickness and shear modulus must be adequate.

Stiffness

The sandwich panel should have sufficient shear and bending stiffness to prevent excessive deflection.

Shear crimping

To prevent the core from prematurely failing in shear under compression loads the core thickness and shear modulus must be adequate.

Intra cell buckling

For a given covering material, the core cell size need to be small enough to prevent intra cell buckling.

Skin wrinkling

To prevent a skin wrinkling failure the compressive modulus of the facing cover and the core compression strength both should be high enough.

Local compression

To resist local loads on the panel surface the core compressive strength should be adequate

1.6 Structural Core Material:

There are variety of core material available in market also many of them claim to be suitable as structural core material. There is no valid definition or line between pure insulation core material and structural core material, but anything that carries load can be treat as a structural core material. The main purpose of designer in case of selecting core material should be core material will not fail at any applied load and there is no deformation in core thickness, thus it requires a high Young’s modulus perpendicular to face layers. The core layer is exposed to shear so shear strain in the core is main cause of global deformation and shear stress. The core material and thickness of core are two main factors that select the most of the properties of the sandwich structure. The core material should have following features:

- Thermal insulation.
- Shear strength and Shear modulus.
- Dampening of vibration and noise.
- Low density.
- Stiffness perpendicular to the faces.

As the core in sandwich structures a variety of materials find application:

Polymeric foam cores: A foams are commonly used use as core material. They are generally closed or open cells. Different variety of polymeric foam are available with different physical properties some of them are as follow:

Polyvinylchloride (PVC): For high performance structure PVC is used as core material. It comes in two form rigid and flexible. PVC are produced by polymerization of the vinyl chloride monomers. They have high hardness and elastic modulus can vary between 1500-3000MPa.

Syntactic cores: In a resin matrix it consist of hollow spheres. These hollow spheres is nothing but normally polymeric or glass. These are lightweight composite.

1.6.1 Magneto-rheological fluid:

Rheology means science of material flow under external load condition so magneto rheology means fluid in which viscosity increases with application of magnetic field to a point it become viscoelastic solid. For many industrial applications Magneto rheological (MR) fluid technology has been proven, like seat dampers, shock absorbers, actuators, etc. However, MR fluids can only show a yield stress of 50–100 kPa at a magnetic field of 150–280 kA/m. So, the appropriate choice of various MR fluid components like carrier fluid, magnetic particles and additives to fulfill the high yield stress applications.

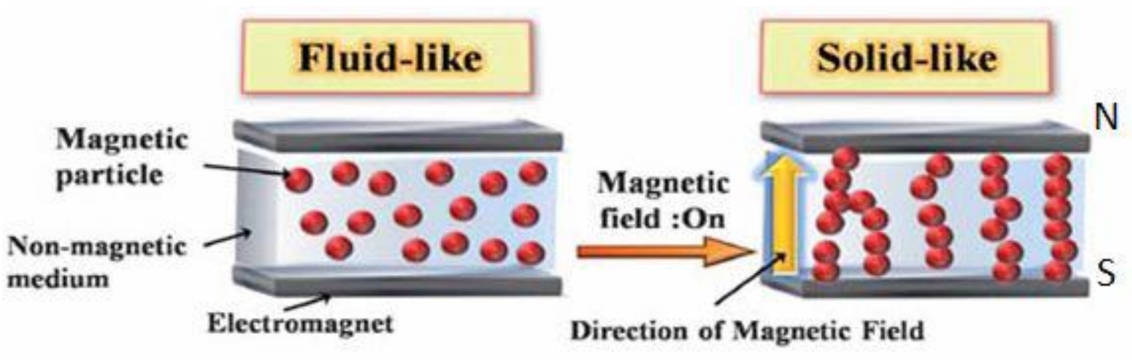

Figure 2 Structural change in MR Fluid

For an MR fluid, the yield stress can be increased or decreased with the strength of the magnetic field.

$$\tau = \tau_{y(H)} + \mu_p \dot{\gamma}$$

Where,

$\tau_{y(H)}$ Is known as yield stress due to applied magnetic field H. μ_p is plastic viscosity and $\dot{\gamma}$ is rate of shear strain.

Depending on the type of requirement employed like deformation MR fluids can be operated in three working modes such as squeeze mode, shear mode and valve mode [27, 28]. In case of the squeeze mode, the MR fluid is squeezing by a normal pressure in the direction of the magnetic field under dynamic or static (compression or tension) loadings. In shear mode, the MR fluid is situated between the two surfaces moving in relative to each other while the magnetic field flows normal to the direction of motion of shear surfaces. While in case of valve mode, between static plates the MR fluid is forced to flow directly.

1.6.2 Components of MR Fluid:

As per need of the application formulation of MR fluid depends. The following basic component contains the MR fluid [29].

MRF Components

Liquid Carrier	Magnetic Particles	Additives	Surfactants
• Mineral Oil • Silicone Oil • Glycol • Water	• Electrolytic Iron • Iron/Cobalt alloys • Nickel Alloys	• Grease • Lithium Stearate • Arsil 1100	• Oleic Acid • Soy lecithin • Citric Acid

Table 1 Component of MR Fluid

1.6.3 Selection Criteria for MR Fluid Components:

The modification in one or more components influences the MR effect. The various selection criteria for MR fluid components are given below

Liquid carrier:

Carrier liquid is the major constituent of MR fluids 60-80 per cent by volume. For highest magneto rheological effect the viscosity of liquid carrier should be less and it should be independent of temperature.

Silicone Oil

Silicone oil has good heat-transfer and good temperature-stability characteristics, very low vapour pressure, oxidation resistance and high flash points. Silicone oil is very hard to seal. Over a wide temperature span there is little change in physical properties and serviceability from -40 to 204°C and a relative flat viscosity temperature slope.

Magnetic Particles:

The size of magnetic particles is near to the order of 1μm to 10μm. The concentration of magnetic particles in base fluid can go up to 50%. The attainable force increases as the size of magnetic particle increases but at the cost of increased off state viscosity of MR fluid [30]. High saturation magnetization, Low coercively, high permeability, small hysteresis loop and small remnance are other characteristics of magnetic materials used for the formulation of MR fluid.

Additives

Grease or other thixotropic additives having highly viscous materials are used to improve settling stability. Magnetic particles are coated with some materials like gaur gum, polystyrene (PS) etc. to reduce the CI particles density, to enhance the sedimentation stability and to avoid the CI particles from coming in contact with each other.

1.7 Face Material:

The top and bottom layer of sandwich structure is known as Face material. As layer is in sheet form. The top and bottom layers face materials transmit the tensile and compressive stresses in the sandwich. From any structural materials that are available in the form of thin sheets can be used as a face material. Many times flexural rigidity is very small and can be ignored sometimes. Commonly used face material is Fiber glass-reinforced plastics and acceptable for use.

- High tensile and compressive strength
- Wear resistance
- High stiffness giving high flexural rigidity
- Resistance to different conditions (chemical, heat, etc.)
- High impact resistance
- Good surface finish

1.8 Applications of Sandwich Structure:-

- Naval ships
- Automotive Industry
- Refrigeration
- Aerospace arena
- Rail Industry

1.9 Design Consideration:-

During its design life a sandwich structure should be capable of taking structural loads such as tensile, compressive, shear etc. In addition, structural integrity should be maintain during in-service environments. The following criteria should be satisfied by structure:

- The face sheets when subjected to applied load then it should be sufficiently stiffed to take tensile, compressive and shear load.
- To prevent the overall buckling of column or sandwich beam MR fluid should have sufficient shear modulus.
- To prevent the wrinkling of the face sheets under applied loads stiffness of the MR fluid and compressive strength of the face sheets should be sufficient.
- To prevent inter-cell buckling of the face sheets under design loads the core cells should be small enough.
- To withstand shear stresses MR fluid should have sufficiently stiff.
- Structural integrity should be maintained during in-service environment by face sheet, MR fluid and adhesive material.
- To prevent excessive deflections under applied loads the sandwich structure should have sufficient flexural and shear rigidity.
- To prevent crushing due to applied loads acting normal to the face sheets or by compressive stresses produced by flexure the core shall have sufficient compressive strength.
- When sandwich beam subjected to vibration there should not be any leakage of MR fluid.

Chapter No. 2

Literature Review

2.0 <u>LITERATURE REVIEW</u>

To discuss dynamic stiffness model Banerjee et al.[3] uses unequal thickness of three layered sandwich beam and it was used for investigating free vibration characteristics with the help of using Timoshenko beam theory they established their layers and By relating amplitude of harmonic varying load they developed a dynamic stiffness matrix.

Qing Sun (4) he studied dynamic characteristics of an adaptive beam with MR fluid. Using oscillatory rheometry technique the relation between complex shear modulus of MR fluid within pre yielding regime and magnetic field is studied. By using different magnetic field he studied vibration characteristics of adaptive beam.

Zekeriya Parlak, Ismail Calli, Tahsin Engin [5] find out the ideal design of MR damper by using finite element analyses. Electromagnetic analysis of magnetic field, CFD analysis of MR flow and Finite element method has been used to obatin the optimal value of design parameters. With experimental study the optimal design of MR damper obtained has been verified. Also they manufactured and test the dampers.

Vikram G Kamble, Samson Paul Pinto, Sagar G Kamble[6] uses simply supported beam subjected to testing and analysis under both damped and un-damped conditions. The variations in the various dynamic parameters like vibration amplitudes, natural frequencies and damping factors were observed.

Yalcintus[7] examined by applying magnetic field level outwardly over the MR Fluid layer. The stiffness and damping will be varied with magnetic field, these changes in the stiffness and damping properties would be used to improve the vibration characteristics of adaptive beams such as vibration amplitudes, natural frequency, loss factor and mode shape.

To calculate the complex shear modulus of the MR Fluid a free oscillation experiment was performed. Optimal shapes of a partially treated MR sandwich beam to achieve maximum model damping factor conforming to the first five flexural mode either individually or simultaneously done by Vasudevan Rajamohan [8].

For the effect of viscoelastic layers Ditoranto [9] has derived auxiliary equation. For solving static and dynamic bending problems use of this equation with the ordinary bending equation formed for homogeneous beams. They formed the homogeneous, six orders and complex differential equation of the finite length viscoelastic layered beam and determined the loss factors and natural frequencies for the freely vibrating beam.

By decoupling the sixth order equation Mead [10] extended the Ditoranto work and for the forced vibration analysis he modeled the sixth order equation of motion in terms of transverse displacements in a three layered sandwich beam with viscoelastic core and when the beam is excited by the damped normal loads complexity in mode would exist which is proportional to the transverse inertia loading on the beam.

The vibration damping in the four layered sandwich beam was discussed by Yadav [11]. For obtaining the equation of motion for the vibration analysis he used the method of beam theory and equilibrium forces. The analysis conducted for the simply supported boundary conditions with the mass and rubber spring mounted on a sandwich beam structure.

For nonlinear vibrations of sandwich beams Jacques et al. [12] modeled a zigzag model with help of FEA to describe the displacement field and studied the effect of amplitude on the damping properties of sandwich beams. The hereditary integrals with complex modulus handled the behavior of viscoelastic beam.

To know the damping characteristics of the thin and thick viscoelastic core beams Mohammadi and Sedaghati [13] studied the semi-analytical finite element method. They also investigated effect of imperfect bonding with in the layers. To solve the eigenvalue problem they developed an efficient algorithm due to the frequency dependent properties in viscoelastic material.

The passive damping systems were focused by Barbosa et.al [14] on viscoelastic materials in the laminated places. To characterizing the viscoelastic materials Golla Hugles Method (GHM) has been used and finite element model based on GHM has been presented and validated with the various classic formulation and numerical comparisons.

Using the elementary theory for the nonlinear vibrations Daya [15] modeled the viscoelastic sandwich beam. For coupling the harmonic balance Galerkin analysis was used and discussed about the effects of the boundary conditions and temperature on the vibration response.

Using the dynamic stiffness theory Banerjee et al. [16] developed a 3-layered sandwich beam for calculating the free vibration characteristics. They supposed the top layer and bottom layers behave like a Rayleigh beams, while the core layer behave like a Timoshenko beam for the harmonic analysis. They discussed the mode shapes and natural frequencies of various problems. They developed equation which found in exact with analytical form.

For vibration analysis and damping characteristics Mohammadi et al. [17] uses sandwich cylindrical structures investigated for various boundary conditions. To cover the untreated portions he used the electrorheological fluids in the unconstrained viscoelastic material. The results indicated that the sandwich beam treated partially with the electrorheological fluids provides improved damping performance than fully treated for various boundary condition.

Considering the non-linear displacement field in the viscoelastic layer Barber et al. [18] obtained the finite element model for the 3 layer viscoelastic sandwich beam. They used the approximation that the variable of viscoelastic core layers are to be expressed in the top and bottom layer displacement field and at resonant driving frequencies he predicted the displacements and compared with the literature having the test data available.

For performing the vibration analysis of a 3-layered sandwich beam consists of spring mass Khalili [19] used the finite element formulation and dynamic stiffness method. Also some numerical examples are used for deliberating the finite element formulation and dynamic stiffness matrix.

Using the linear and nonlinear displacement at its core layer Grewel et al. [20] have showed a sandwich beam with help of the finite element method. To find out the effect of core layer thickness Parametric studies were carried out to find the loss factor and the natural frequencies for the sandwich beam structure and they considered the partial treatment of the structure to obtain the more damping for the fixed free and fixed -fixed boundary conditions.

By using arbitrary condition Sakihama et al. [21] established a method for studying the free vibration of the sandwich beam with elastic and viscoelastic layer. The Green function is used to obtain the characteristic equation for free vibration which describes from the discrete solution of governing differential equation. The behavior of sandwich beam was studied by using characteristic equation with the trial and error approach for free vibration.

A modeling technique for multi layered viscoelastic sandwich beams are done by Fei Lin and Mohan [22] to obtain the vibroacoustics. By using Biot damping in the model they provide the non-linear behavior for core layer. The FEA method is used for the vibration investigation of the multilayered sandwich beam.

For the three layered constrained sandwich beam Won et al. [23] observed the problem with the markus and mead two sets of differential equation of motion, to resolve that they take constrained three layered sandwich structures and symmetric straight damped are derived using the virtual strain energy and kinetic energy are mentioned in terms of the transverse shear strain and axial displacement of a viscoelastic core layer. NASTRON 3D-solid element was used to compared and validate their model.

The free vibration analysis of a sandwich beam by using FGM as a core material is discussed by Amirani et al. [24]. They constrained the Galerkin method and formulation for 2-dimensional elastic plastic problems. At last, by using the finite element analysis they got the first ten natural frequencies.

Using piezoelectric layers Yazhuk [25] developed the sandwich beam and joined with the electro-mechanical forced vibration. In nonlinear behavior they investigated the effect of passive layer on the beam. Using the full model with the approximate model they obtained the comparison between the calculated transient responses of the beam.

For the dynamic and static analysis Bekuit et al. [26] considered the quasi-2 dimensional finite element formulation. The model is consist of three layers with longitudinal and transverse displacement field. The formulations are independent of flexibility of the core layer.

Euler Bernoulli beam theory was used by Chen et al. [27] for governing the equation of motion for the system. By applying mass at free end of cantilever beam he analyzed resonant frequency and loss factor for system. They define that physical properties and geometric properties are main cause of variation in resonant frequency and loss factor of constrained viscoelastic structure.

Kerwin [28] presented constrained viscoelastic layers with effective damping and parameter such as wavelength of bending waves, elastic moduli and thicknesses are cause an effect on damping and estimated complex shear modulus of structure and heat dissipation occurred through shear phenomenon. By neglecting the boundary condition damping factors have determined experimentally for a number of constraining layers

The study of effect of viscoelastic adhesive layer on damping of structure is done by Bai and Sun [29] and newly developed sandwich beam theory is used to find dynamic response of the structure. For achieving the accurate kinematics he molded the core layer of the flexible viscoelastic with the new nonlinear displacement field. The properties of viscoelastic were supposed to be in a complex modulus which is a function of frequency for a given temperature. They obtained the loss factors and storage modulus for the simply supported beam under harmonic loading and also achieved for all the set of frequencies, results for driving point impedance which are almost matched with the available data reported by Lu and Douglas.

Lu and Douglas [30] the objective of their work is to compare the experimental and analytical forced vibration response of the three layered damped laminate in a format of mechanical impedance to verify the analytical information given by the Mead and Markus They used Mead and Markus analytical model, At mid span they got mechanical impedance for the damped laminated beam with free-free boundary condition with a sinusoidal transverse force.

By using the finite element model Mace [35] modeled the viscoelastic sandwich beams in the layer wise displacement field for knowing dynamic behavior. The model developed was in 3D and it is very problematic and expensive for the implementation and it applicable for very thin layer of core of sandwich structure. It also difficult to make mesh for analysis.

Chapter No. 3

Mathematical Formulation

3.0 MATHEMATICAL FORMULATION:

Figure shows a three layered sandwich beam having a length of L. Any of the boundary conditions can be applied to beam. The following assumptions are considered to develop finite element model:

- There is perfect continuity at the interfaces and no slip occurs between the face layers and core material
- For sandwich beam, linear theory of elasticity and viscoelasticity are used.
- The shear deformation and rotary inertia of constraining layer is negligible.

Elemental matrices:

Figure 4 shows the element model of sandwich beam consists of two nodes for each face layer and each node has three degrees of freedom. Nodal displacement is as follow,

$$\{\Delta^e\} = \{u_{1i}\ u_{3i}\ w_{1i}\ w_{3i}\ \theta_{1i}\ \theta_{3i}\ u_{1j}\ u_{3j}\ w_{1j}\ w_{3j}\ \theta_{1j}\ \theta_{3j}\}^T \tag{1}$$

Where i and j are element nodal numbers. The transverse displacement, axial displacement and rotational angle of the constraining layer can be expressed in terms of nodal displacements and finite element shape functions given as,

$$u_1 = [Nu_1]\ \{\Delta^e\},\ u_3 = [Nu_3]\ \{\Delta^e\},$$

$$w_1=[N_{w1}]\{\Delta^e\}, w_3=[N_{w3}]\{\Delta^e\},$$

$$\theta_1=[N_w]'\{\Delta^e\}, \theta_3=[N_{w3}]'\{\Delta^e\} \tag{2}$$

Also, the shape functions are given by

$$[Nu_1] = [1-\xi\ 0\ 0\ 0\ 0\ 0\ \xi\ 0\ 0\ 0\ 0\ 0]$$

$[N_{u3}] = [0\ 1-\xi\ 0\ 0\ 0\ 0\ 0\ \xi\ 0\ 0\ 0\ 0]$

$[N_{w1}] = [\ 0\ \ 0\ \ (1-3\xi^2+2\xi^3)\ \ 0\ (\xi-2\xi^2+\xi^3)L_e\ \ 0\ \ 0\ \ 0\ 3\xi^2-2\xi^3\ \ 0\ (-\xi^2+\xi^3)\ L_e\ 0]$

$[N_{w3}] = [\ 0\ \ 0\ \ 0\ (1-3\xi^2+2\xi^3)\ \ 0\ (\xi-2\xi^2+\xi^3)L_e\ \ 0\ \ 0\ \ 0\ 3\xi^2-2\xi^3\ \ 0\ \ (-\xi^2+\xi^3)L_e]$

Where

$\xi = \frac{x}{L}$ L is the length of the element.

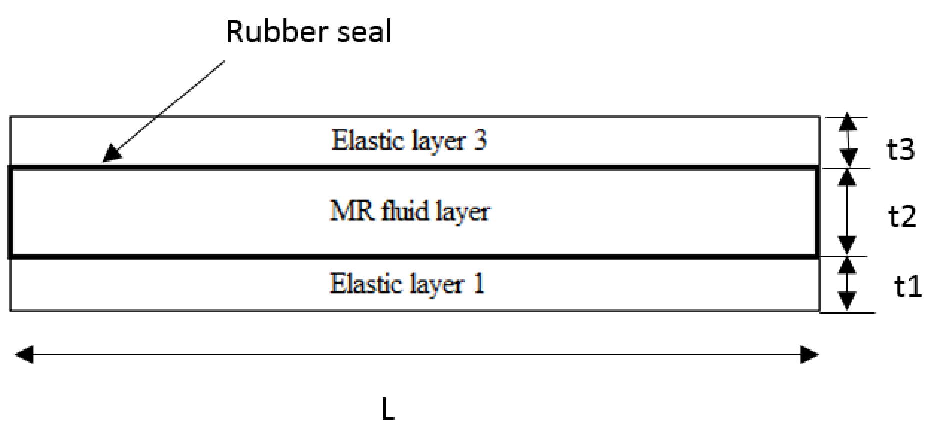

Figure 3Sandwich Beam with MR core

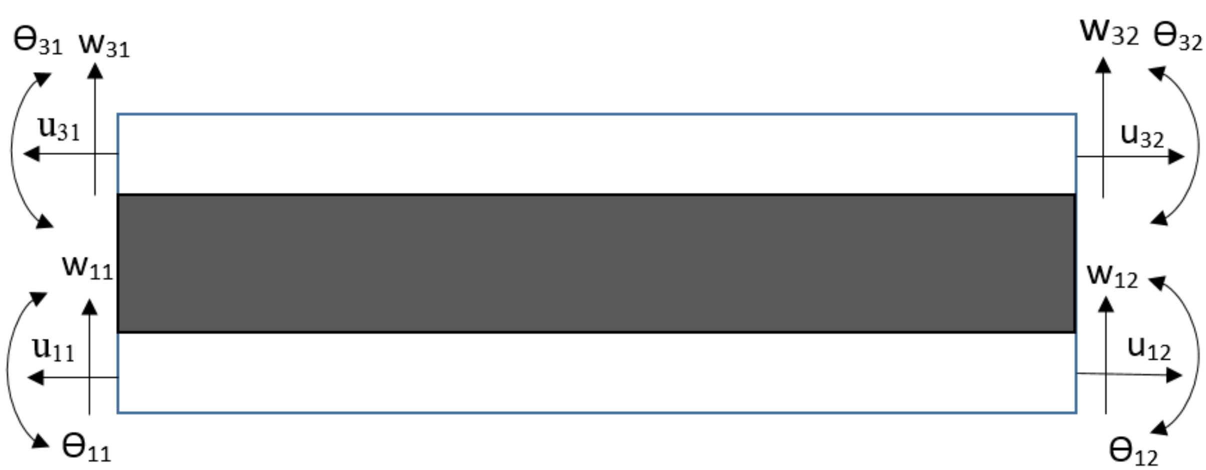

Figure 4 Finite element model for layered sandwich beam

The potential energy ($U^{(e)}$) of the element is equal to addition of potential energy of the face layers and viscoelastic layers.

$$U^{(e)} = U_k^{(e)} + U_v^{(e)} \tag{3}$$

3.1 Element Matrices of constraining layers:

The potential energy of face layer is given as,

$$U_k^{(e)} = \frac{1}{2}\int_0^l E_k I_k \left[\frac{d^2 w_k}{dx^2}\right]^2 dx + \frac{1}{2}\int_0^l E_k A_k \left[\frac{du_k}{dx}\right]^2 dx \qquad k = 1,3 \tag{4}$$

Where, the notations 1 and 3 represents the lower and upper face layers. I, E and A are the moment of inertia Young's modulus and cross-sectional area respectively.

The kinetic energy of face layer is given by,

$$T_k^{(e)} = \frac{1}{2}\int_0^l \rho_k A_k \left[\frac{dw_k}{dt}\right]^2 dx + \frac{1}{2}\int_0^l \rho_k A_k \left[\frac{du_k}{dt}\right]^2 dx \qquad k = 1,3 \tag{5}$$

Where, ρ is the mass density.

By putting equation (2) in to equation (4) and equation (5) , the element kinetic and potential energy of the constraining layers can be given as,

$$U_k^{(e)} = \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T \left(\left[K_{ku}^{(e)}\right] + \left[K_{kw}^{(e)}\right]\right)\{\Delta^{(e)}\} \qquad k = 1,3 \tag{6}$$

and

$$T_k^{(e)} = \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T \left(\left[M_{ku}^{(e)}\right] + \left[M_{kw}^{(e)}\right]\right)\{\Delta^{(e)}\} \qquad k = 1,3 \tag{7}$$

Where,

$$\left[K_{ku}^{(e)}\right] = \left[K_{1u}^{(e)}\right] + \left[K_{3u}^{(e)}\right]$$

$$\left[K_{ku}^{(e)}\right] = E_1 A_1 \int_0^l [N_1]^T [N_1] dx + E_3 A_3 \int_0^l [N_3]^T [N_3] dx$$

$$\left[K_{kw}^{(e)}\right] = \left[K_{1w}^{(e)}\right] + \left[K_{3w}^{(e)}\right]$$

$$\left[K_{kw}^{(e)}\right] = E_1 I_1 \int_0^l [N_w]''^T [N_w]'' dx + E_3 I_3 \int_0^l [N_w]''^T [N_w]'' dx \qquad (8)$$

$$[M_{ku}^e] = [M_{1u}^e] + [M_{3u}^e]$$

$$[M_{ku}^e] = \rho_1 A_1 \int_0^l [N_1]^T [N_1] dx + \rho_3 A_3 \int_0^l [N_3]^T [N_3] dx$$

$$\left[M_{kw}^{(e)}\right] = \left[M_{1w}^{(e)}\right] + \left[M_{3w}^{(e)}\right]$$

$$\left[M_{kw}^{(e)}\right] = \rho_1 A_1 \int_0^l [N_w]^T [N_1] dx + \rho_3 A_3 \int_0^l [N_w]^T [N_3] dx$$

The dot denotes differentiation with respect to time t.

3.2 Element Matrices of viscoelastic layer:

(a) The potential energy of the Viscoelastic layer:

The potential energy of Viscoelastic layer due to shear, longitudinal and transverse deformation is given by,

$$\mathrm{U}_{\mathrm{v}}^{(\mathrm{e})} = \frac{1}{2}\int_0^{\mathrm{l}} \bar{G}\mathrm{A}_{\mathrm{v}}\gamma_{\mathrm{v}}^2\mathrm{dx} + \frac{1}{2}\int_0^{\mathrm{l}} \mathrm{E}_{\mathrm{v}}\mathrm{A}_{\mathrm{v}}\varepsilon_{\mathrm{vL}}^2\mathrm{dx} + \frac{1}{2}\int_0^{\mathrm{l}} \bar{G}\mathrm{A}_{\mathrm{v}}\varepsilon_{\mathrm{vT}}^2\mathrm{dx} \tag{9}$$

For sealing of MR fluid a silicone gel is applied at the edges having uniform thickness to hold the MR fluid within the beam. The middle layer of the sandwich beam contains the silicon rubber seal and the Magneto Rheological fluid, however, it is considered as homogenous material layer having equivalent shear modulus and can be represented by moduli and width of two material such as,

$$\bar{G} = G_r\left(\frac{b_r}{b}\right) + G^*(1 - \frac{b_r}{b})$$

Where, *Gr* and *G** are the shear modulus of the rubber and Magneto Rheological fluid respectively. $\overline{\mathrm{G}}$ is the equivalent shear modulus of core [36], b_r and *b* are the widths of the sealing rubber and total beam width respectively,. In the pre yield region, the Magneto Rheological material shows viscoelastic behavior, which can be represented in terms of the complex modulus *G** and given by,

$$G^* = G' + iG''$$

Where, G' is storing modulus of the Magneto Rheological fluid, which represent during a deformation cycle average energy stored per unit volume of the material, and G'' is the loss modulus, it represent the energy losses per unit volume of the material over a cycle.

The simulation is performed for a sandwich beam with MR core with a dimensions of elastic layer is 300mm×30mm×1mm with an identical thickness of Magneto Rheological fluid layers. The various properties of material are,

E_1=E_3=68GPa;$\rho_1 = \rho_3 = 2700\mathrm{kg/m^3}$; $\rho_2 = 3500\mathrm{kg/m^3}$; $\rho_r = 1233\mathrm{kg/m^3}$;

There exist a relation between a shear modulus and applied magnetic field and is given by,

$$G'(B) = -3.3691B^2 + 4.9975 \times 10^3\ B + 0.893 \times 10^6$$

$$G''(B) = -0.9B^2 + 0.8124 \times 10^3\ B + 0.1855 \times 10^6$$

Where, B is known as applied magnetic field in gauss (G). The shear strain γ_v, transverse strain (ε_{vT}) and longitudinal strain (ε_{vL}) due to thickness deformation of the viscoelastic layer from kinematic relationship between the constraining layers is expressed as follows:

$$\gamma_v = \frac{u_1 - u_3}{2} + \frac{(t_1 + t_2)}{2t_2}\frac{\partial w_1}{\partial x} + \frac{(t_2 + t_3)}{2t_2}\frac{\partial w_3}{\partial x}$$

$$\varepsilon_{vT} = \left(\frac{w_1 - w_3}{t_2}\right) \tag{10}$$

$$\varepsilon_{vL} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial x}\right) + \frac{t_1}{4}\left(\frac{\partial w_1}{\partial x}\right) - \frac{t_3}{4}\left(\frac{\partial w_3}{\partial x}\right)$$

Substituting Eq. (2) in to Eq. (10) γ_v,ε_{vL}, ε_{vT} and u_v can be represented in terms of nodal displacements and element shape functions:

$$\gamma_v = [N_\gamma]\{\Delta^{(e)}\}$$

$$\varepsilon_{\mathrm{vL}} = [N_L]\{\Delta^{(e)}\} \tag{11}$$

$$\varepsilon_{\mathrm{vT}} = [N_T]\{\Delta^{(e)}\}$$

Where,

$$[\mathrm{N}_{\gamma}] = \frac{([\mathrm{N}_{\mathrm{u1}}] - [\mathrm{N}_{\mathrm{u1}}])}{t_2} + \frac{(\mathrm{t}_1 + \mathrm{t}_2)}{2\mathrm{t}_2}[\mathrm{N}_{\mathrm{w1}}] - \frac{(\mathrm{t}_3 + \mathrm{t}_2)}{2\mathrm{t}_2}[\mathrm{N}_{\mathrm{w3}}]$$

$$[\mathrm{N}_{\mathrm{L}}] = \frac{1}{2}([\mathrm{N}_{\mathrm{u1}}]' + [\mathrm{N}_{\mathrm{u3}}]') + \frac{\mathrm{t}_1}{4}[\mathrm{N}_{\mathrm{w1}}]'' - \frac{\mathrm{t}_3}{4}[\mathrm{N}_{\mathrm{w3}}]'' \tag{12}$$

$$[\mathrm{N}_{\mathrm{T}}] = \frac{1}{\mathrm{t}_2}([\mathrm{N}_{\mathrm{w1}}] - [\mathrm{N}_{\mathrm{w3}}])$$

Putting eq. (11) in to eq. (9) the potential energy of the viscoelastic layer is given by

$$\mathrm{U}_{\mathrm{v}}^{(\mathrm{e})} = \frac{1}{2}\{\Delta^{(\mathrm{e})}\}^{\mathrm{T}}\left(\left[\mathrm{K}_{\mathrm{v}\gamma}^{(\mathrm{e})}\right] + \left[\mathrm{K}_{\mathrm{vL}}^{(\mathrm{e})}\right] + \left[\mathrm{K}_{\mathrm{vT}}^{(\mathrm{e})}\right]\right)\{\Delta^{(\mathrm{e})}\} \tag{13}$$

Where,

$$\left[\mathrm{K}_{\mathrm{v}\gamma}^{(\mathrm{e})}\right] = \mathrm{G}_{\mathrm{v}}\mathrm{A}_{\mathrm{v}}\int_0^{\mathrm{l}}[\mathrm{N}_{\gamma}]^{\mathrm{T}}[\mathrm{N}_{\gamma}]\mathrm{dx}$$

$$\left[\mathrm{K}_{\mathrm{vL}}^{(\mathrm{e})}\right] = \mathrm{E}_{\mathrm{v}}\mathrm{A}_{\mathrm{v}}\int_0^{\mathrm{l}}[\mathrm{N}_{\mathrm{L}}]^{\mathrm{T}}[\mathrm{N}_{\mathrm{L}}]\mathrm{dx} \tag{14}$$

$$\left[\mathrm{K}_{\mathrm{vT}}^{(\mathrm{e})}\right] = \mathrm{E}_{\mathrm{v}}\mathrm{A}_{\mathrm{v}}\int_0^{\mathrm{l}}[\mathrm{N}_{\mathrm{T}}]^{\mathrm{T}}[\mathrm{N}_{\mathrm{T}}]\mathrm{dx}$$

(b) Kinetic energy of the viscoelastic layer:

$$T_v^{(e)} = \frac{1}{2}\int_0^l \rho_v A_v \left\{\left[\frac{\partial w_v}{\partial t}\right]^2 + \left[\frac{\partial u_v}{\partial t}\right]^2\right\} dx \tag{15}$$

Where ρ_v is the mass density of viscoelastic layer and A_v is the cross-sectional area. The axial and lateral displacement u_v and w_v of the viscoelastic layer derived from kinematic relations between the constraining layers is given by:

$$u_v = \frac{u_3 + u_1}{2} + \frac{t_1}{4}\frac{\partial w_1}{\partial x} - \frac{t_3}{4}\frac{\partial w_3}{\partial x} \tag{16}$$

$$w_v = \frac{w_3 + w_1}{2}$$

Putting eq. (2) in to eq. (16) u_v and w_v can be represented in terms of nodal displacements and element shape functions:

$$\left.\begin{aligned} u_v &= [N_{vL}]\{\Delta^{(e)}\} \\ w_v &= [N_{vT}]\{\Delta^{(e)}\} \end{aligned}\right\} \tag{17}$$

Where,

$$\left.\begin{aligned} [N_{vL}] &= \frac{1}{2}([N_{u3}] + [N_{u1}]) + \frac{t_1}{4}[N_{w1}] - \frac{t_3}{4}[N_{w3}] \\ [N_{vT}] &= \frac{1}{2}([N_{w3}] + [N_{w1}]) \end{aligned}\right\} \tag{18}$$

Substituting equation (2) into equation (17) and (15), the kinetic energy of viscoelastic layers is obtain by,

$$T_v^{(e)} = \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T \left(\left[M_{vL}^{(e)}\right] + \left[M_{vT}^{(e)}\right]\right)\{\dot{\Delta}^{(e)}\} \tag{19}$$

Where,

$$\left[M_{vL}^{(e)}\right] = \rho_v A_v \int_0^l [N_{vL}]^T [N_{vL}] dx \tag{20}$$

$$\left[M_{vT}^{(e)}\right] = \rho_v A_v \int_0^l [N_{vT}]^T [N_{vT}] dx$$

3.3 Total potential energy of the element:

$$U^{(e)} = \sum_{k=1,3} \frac{1}{2}\{\Delta^{(e)}\}^T \left(\left[K_{(k)u}^{(e)}\right] + \left[K_{(k)w}^{(e)}\right]\right)\{\Delta^{(e)}\} + \frac{1}{2}\{\Delta^{(e)}\}^T \left(\left[K_{v\gamma}^{(e)}\right] + \left[K_{vL}^{(e)}\right] + \left[K_{vT}^{(e)}\right]\right)\{\Delta^{(e)}\}$$

$$U^{(e)} = \frac{1}{2}\{\Delta^{(e)}\}^T [K^{(e)}]\{\Delta^{(e)}\} \tag{21}$$

Where,

$$\left[K^{(e)}\right] = \sum_{k=1,3} \left(\left[K_{(k)u}^{(e)}\right] + \left[K_{(k)w}^{(e)}\right]\right) + \left[K_{v\gamma}^{(e)}\right] + \left[K_{vL}^{(e)}\right] + \left[K_{vT}^{(e)}\right] \tag{22}$$

$\left[K^{(e)}\right]$ Is known as stiffness matrix of element.

3.4 Total kinetic energy of the element:

$$T^{(e)} = \sum_{k=1,3} \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T \left(\left[M_{(k)u}^{(e)}\right] + \left[M_{(k)w}^{(e)}\right]\right)\{\dot{\Delta}^{(e)}\} + \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T \left(\left[M_{vL}^{(e)}\right] + \left[M_{vT}^{(e)}\right]\right)\{\dot{\Delta}^{(e)}\}$$

$$T^{(e)} = \frac{1}{2}\{\dot{\Delta}^{(e)}\}^T [M^{(e)}]\{\dot{\Delta}^{(e)}\} \tag{23}$$

Where,

$$\left[\mathrm{M}^{(e)}\right] = \sum_{k=1,3}\left(\left[\mathrm{M}^{(e)}_{(k)u}\right] + \left[\mathrm{M}^{(e)}_{(k)w}\right]\right) + \left(\left[\mathrm{M}^{(e)}_{vL}\right] + \left[\mathrm{M}^{(e)}_{vT}\right]\right) \tag{24}$$

$\left[\mathrm{M}^{(e)}\right]$ is the mass matrix of element.

3.5 Element geometric stiffness matrix

The work done by axial force *P* on element is given by,

$$W_p^{(e)} = \frac{1}{2}\int_0^l P\left[\left(\frac{\partial w_1}{\partial x}\right)^2 + \left(\frac{\partial w_3}{\partial x}\right)^2\right]\partial x$$

Work done by the axial load can be given by,

$$W_p^{(e)} = \frac{1}{2}\left\{\Delta^{(e)}\right\}^T K_g^{(e)}\left\{\Delta^{(e)}\right\}$$

Where, $[K_g^{(e)}]=\int_0^l \left[N_{w1}\right]'^T\left[N_{w1}\right]' + \left[N_{w3}\right]'^T\left[N_{w3}\right]' \; dx$, the elemental geometric stiffness matrix.

3.6 Governing Equation of motion:

By using Hamilton's principle the element equation of motion for a sandwich beam is obtained by,

$$\delta\int_{t_1}^{t_2}\left(T^{(e)} - U^{(e)} + W_p^{(e)}\right)dt = 0 \tag{25}$$

Substituting equation (21) And (23) in to eq. (25) the equation of motion for the sandwich beam element is obtained by:

$$\left[M^{(e)}\right]\left\{\ddot{\Delta}^{(e)}\right\} + \left[K^{(e)}\right]\left\{\Delta^{(e)}\right\} - P\left[K_g^{(e)}\right]\left\{\Delta^{(e)}\right\} = 0 \tag{26}$$

Assembling mass matrix, elastic stiffness matrix and geometric stiffness matrices of each element, the equation of motion for the beam is given as,

$$[M]\{\ddot{\Delta}\}+[K]\{\Delta\}-P\left[K_g\right]\{\Delta\}=0$$

Governing equation to determine natural frequency

$$[M]\{\ddot{\Delta}\}+[K]\{\Delta\}=0$$

Governing equation to determine buckling load

$$[K]\{\Delta\}-P\left[K_g\right]\{\Delta\}=0$$

Where $\{\Delta\}$ is the global displacement matrix.

Chapter No. 4

Result & Discussion

4.0 Result and discussion:

The sandwich beam is discretized using finite element. A ten element discretization satisfy the convergence requirement. The validation of the present formulation was done by comparing the calculated first three natural frequencies with those of table shown [36],

Magnetic Field Intensity (g)	Boundary Conditions	Natural Frequency(Hz) Mode 1	present	2	present	3	present
0	SSB	104.28	103.5	396.96	395.5	882.36	882
	CFB	42.38	42.23	227.44	226.9	616.92	615.12
	CCB	224.91	223.70	613.11	612.8	1193.10	1192.92
100	SSB	106.72	106	399.54	399.2	884.98	884.17
	CFB	44.78	43.83	230.95	230.1	620.15	620.1
	CCB	226.33	226.10	615.05	614.23	1195.20	1194.5
200	SSB	108.76	108.36	401.73	400.95	887.21	886.95
	CFB	46.65	45.95	233.87	223.15	622.88	622.1
	CCB	227.53	227.1	616.71	616.17	1197.10	1195.96
300	SSB	110.43	109.23	403.54	402.5	889.05	888.43
	CFB	47.42	46.35	235.24	234.68	624.13	622.94
	CCB	228.52	227.06	618.08	617.25	1198.60	1197.63

Table 2 Validation of natural frequencies sandwich beam with different boundary condition.

The variation of the fundamental buckling load parameter (P_b), defined as the ratio of fundamental buckling load of the beam to P^*, with core thickness parameter (t_2/t_1) is shown in Figure 5, for the four boundary conditions considered. It is seen from the figure that for core thickness parameter, 0.5 to 5.0, for all the four boundary conditions there is a linear increase in fundamental buckling load parameter with increase in core thickness parameter. Moreover as expected for any value of thickness ratio the buckling load for fixed-free end condition is minimum, preceded by pinned-pinned, fixed-pinned and fixed- fixed cases. This is due to the fact that more the rigid the end supports are, the more is the critical buckling load.

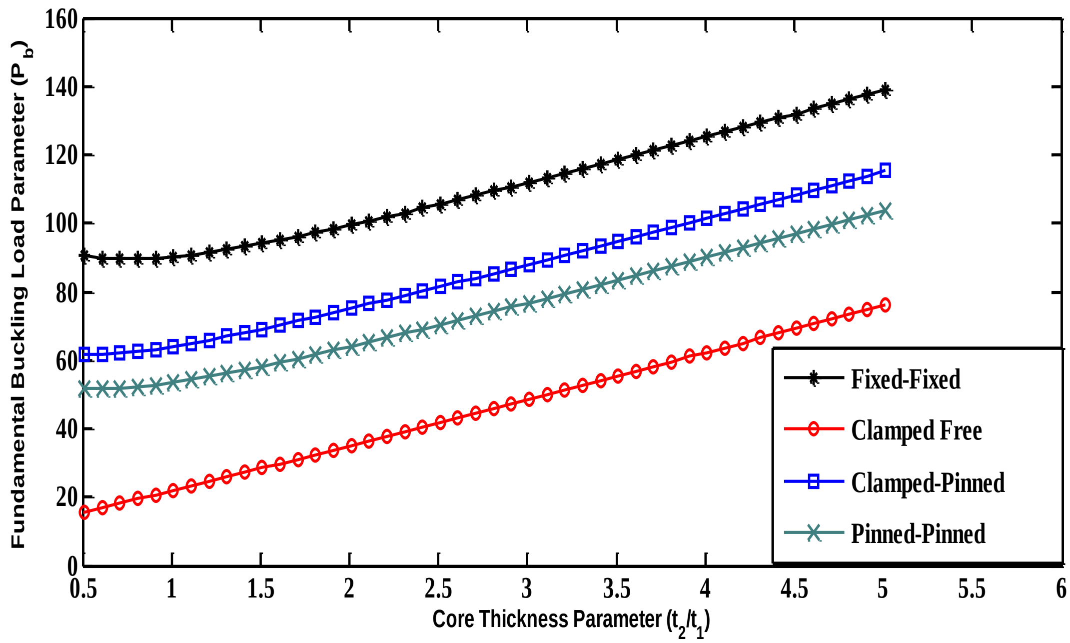

Figure 5 Core thickness parameter Vs fundamental buckling load parameter

Figure 6 shows the effect of core thickness parameter on the fundamental mode frequency parameter (f). The fundamental frequency parameter is defined as the ratio of fundamental frequency of the sandwich beam to ω_o . The variation of fundamental frequency parameter with core thickness parameter shows the similar trend as those for fundamental buckling load for the four boundary conditions. These behaviors are also due the same reasons as mentioned above.

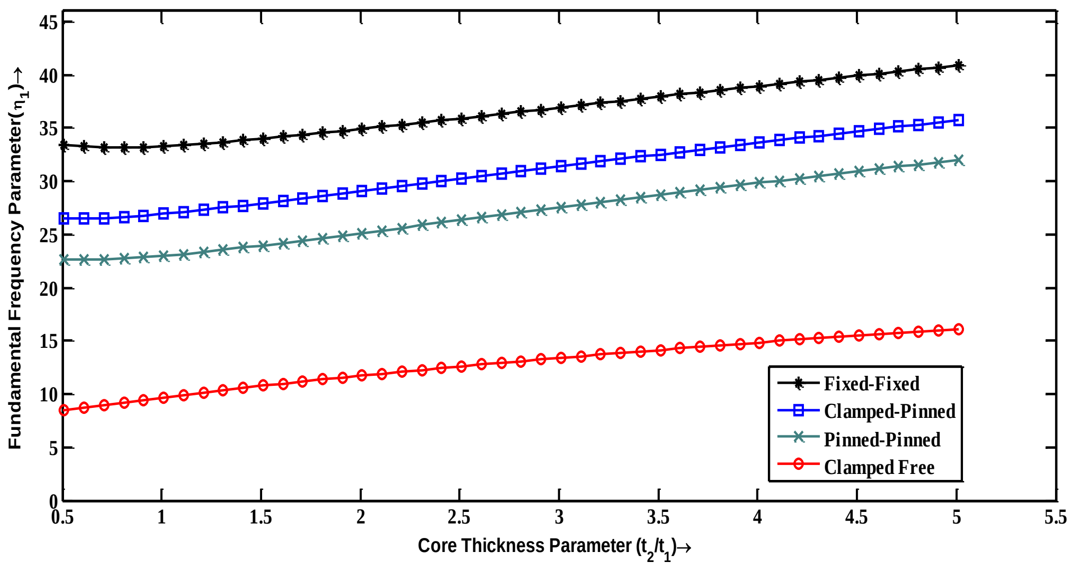

Figure 6 Core thickness parameter Vs fundamental frequency parameter

The variation of the fundamental mode loss factor (η) with the core thickness parameter is shown in Figure 7. For all the four boundary conditions the fundamental loss factor increases with increase in core thickness parameter. When the t_2/t_1 has higher values the shear strain becomes comparatively smaller and hence damping decreases.

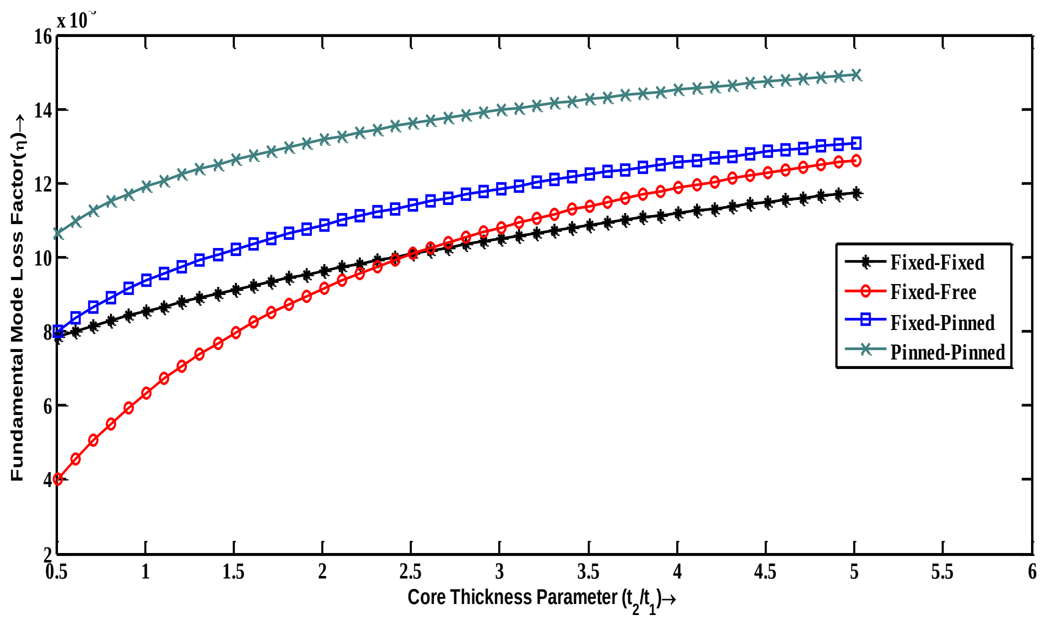

Figure 7 Core thickness parameter Vs fundamental loss factor, magnetic field strength 300G

In Figure 8 the effect of magnetic field strength (G) on the fundamental buckling load parameter is shown for the four boundary conditions. It can be seen that the fundamental buckling load parameter increases with increase in magnetic field strength. The rate of increase in fundamental buckling load parameter is more in higher range of magnetic field strength. This happens due to the fact that with increase in magnetic field strength the stiffness of the MR fluid core layer increases.

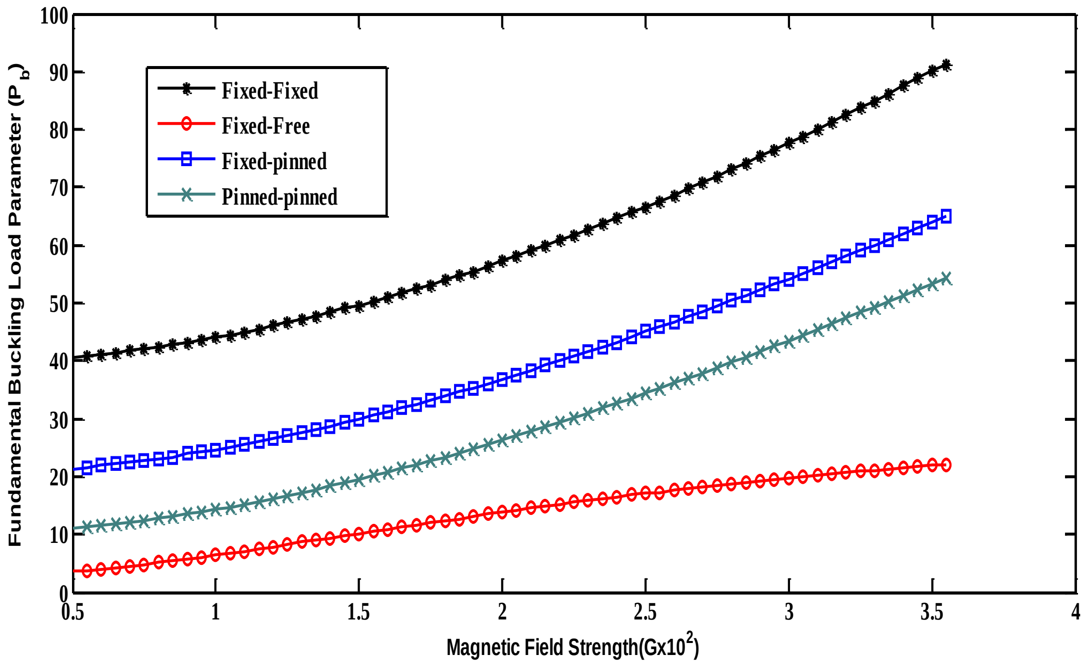

Figure 8 Magnetic field Vs fundamental buckling load parameter at t2/t1=1

Figure 9 shows the variation of fundamental frequency parameter with magnetic field strength. The behavior is same as those for fundamental buckling load. This is also due to increase in the stiffness of the MR fluid core layer with increase in the strength of the magnetic field.

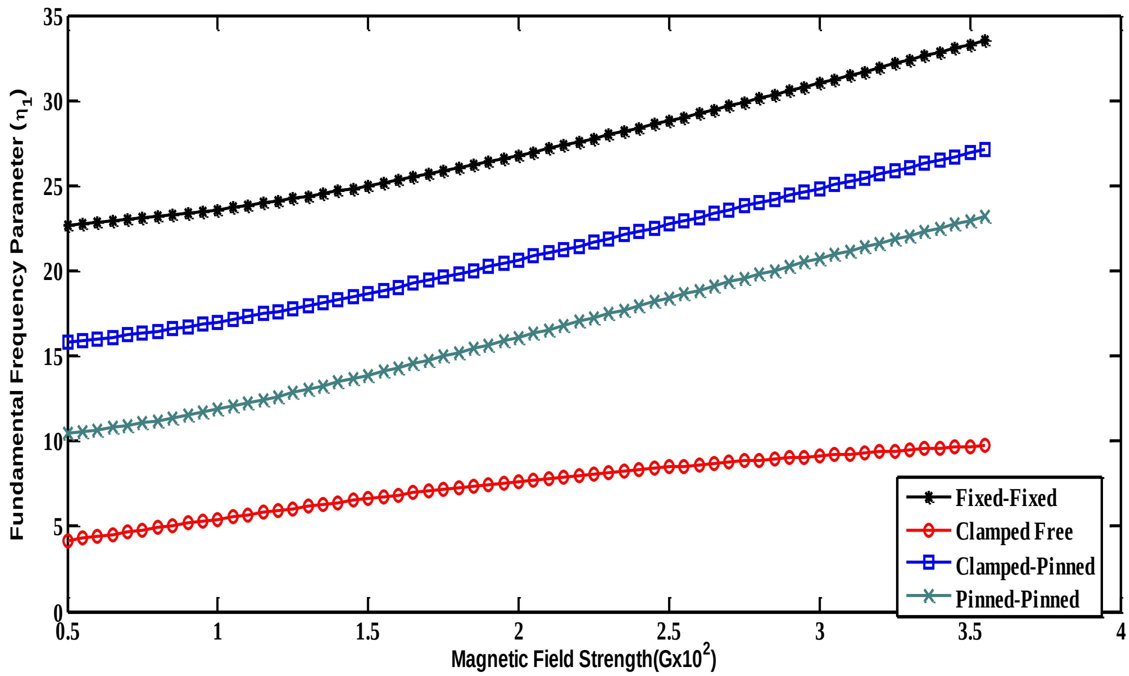

Figure 9 Magnetic field Vs fundamental buckling load parameter at t2/t1=1

Figure 10 shows the variation of fundamental mode loss factor (η) with magnetic field strength (G). It is seen that with increase in magnetic field strength the fundamental loss factor increases and attains its maximum value. Then with additional increase in the field strength the loss factor decreases. The value of E at which maximum loss factor occurs depends on the end conditions. The variation of η with G depends on the correlation between the magnetic field strength and the shear storage modulus and loss modulus. In the range of values of G, for which the shear storage modulus is less than the loss modulus, the loss factor increases with increase in G and for values of G for which the loss modulus is smaller compared to the shear storage modulus the loss factor decreases with increase in G.

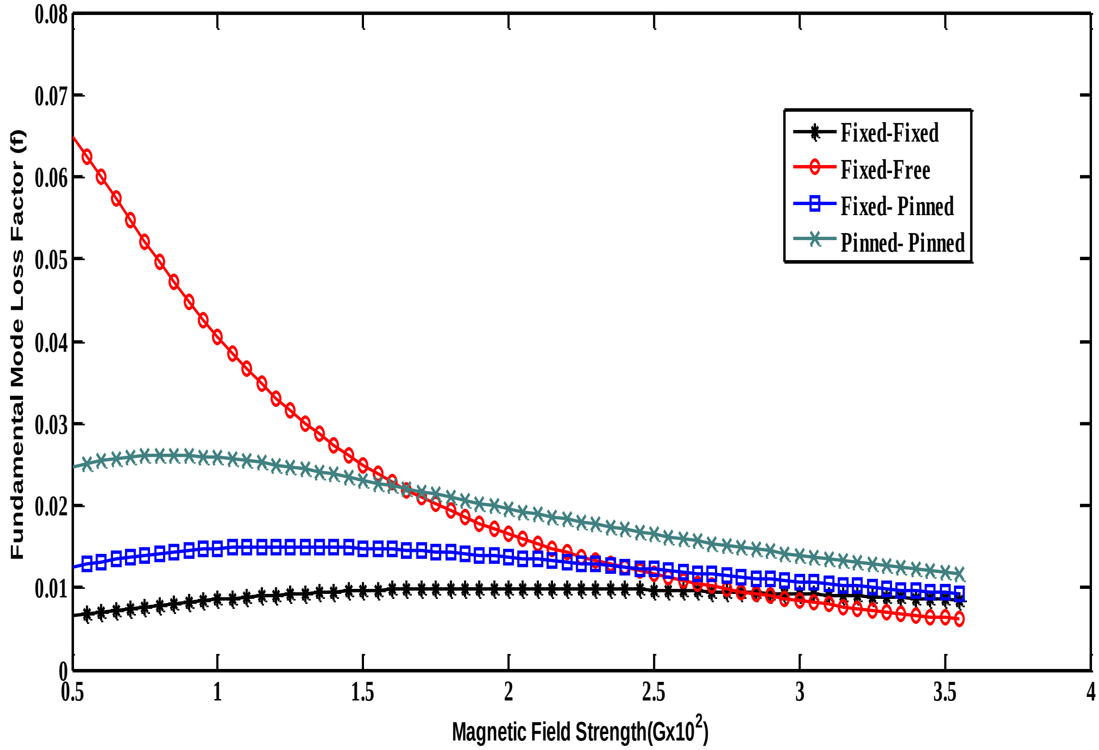

Figure 10 Magnetic field Vs fundamental loss factor, t2/t1=1

Figure 11 shows the effect of shear parameter (g) [37] on fundamental buckling load. The shear parameter is defined as,

$$g = \frac{G'}{\frac{t2}{t1}} \left(\frac{L}{t1}\right)^2 \left(\frac{2}{E1}\right)$$

It can be seen that with increase in shear parameter also the fundamental buckling load parameter increases almost linearly for fixed-fixed, fixed-pinned and pinned-pinned end conditions. For fixed-free end condition though the fundamental buckling load parameter increases with increase in shear parameter the rate of increase is relatively less as compared to the other three boundary conditions, especially at the higher values of g. From the definition of shear parameter (g), increase in its value means, increase in the length of the beam with other geometrical and material parameters remaining constant. So increase in length of the beam leads to increase in buckling load parameter.

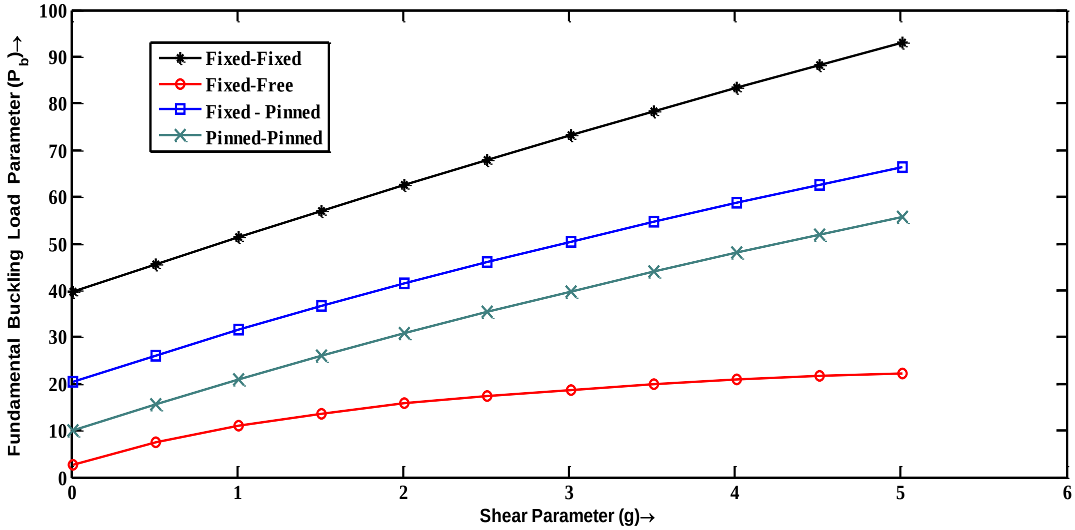

Figure 11 Shear parameter Vs fundamental buckling load parameter, t2/t1=1

Figure 12 shows the effect of shear parameter on fundamental frequency parameter (f). With increase in g, the fundamental frequency parameter increases for all the four boundary conditions. But for fixed-free end condition high values of g does not have significant effect on increase in f. The fundamental frequency parameter increases with length of the beam, which increases with shear parameter.

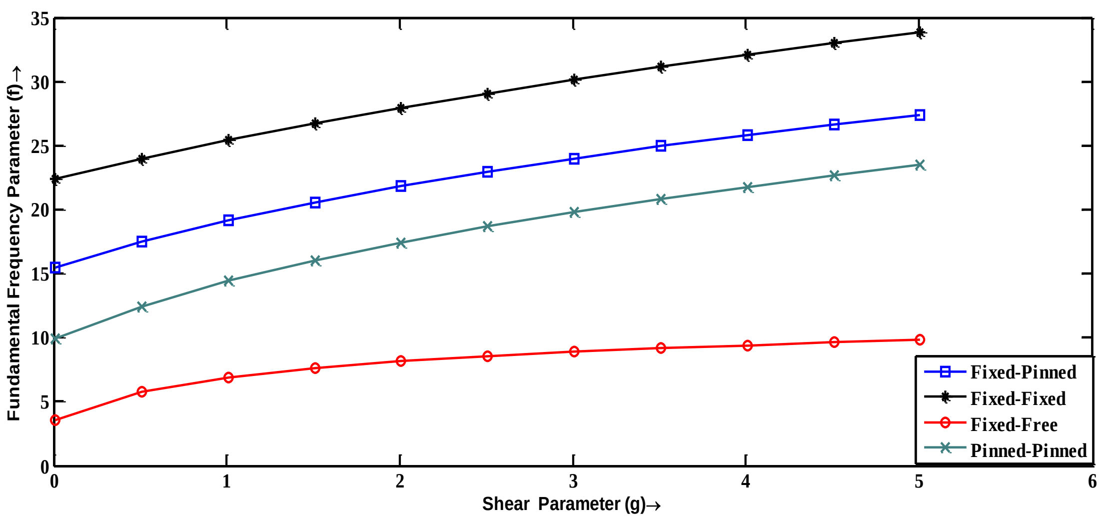

Figure 12 Shear parameter Vs fundamental frequency parameter, t2/t1=1

Figure 13 shows the effect of shear parameter on fundamental mode loss factor (η). For clamped-free end condition the loss factor gets increases with increase in shear parameter and attains its maximum value and with further increase in g, the loss factor decreases. For other three end conditions the system loss factor gets increases with increase in shear parameter. But for higher values of g the effect becomes less dominant.

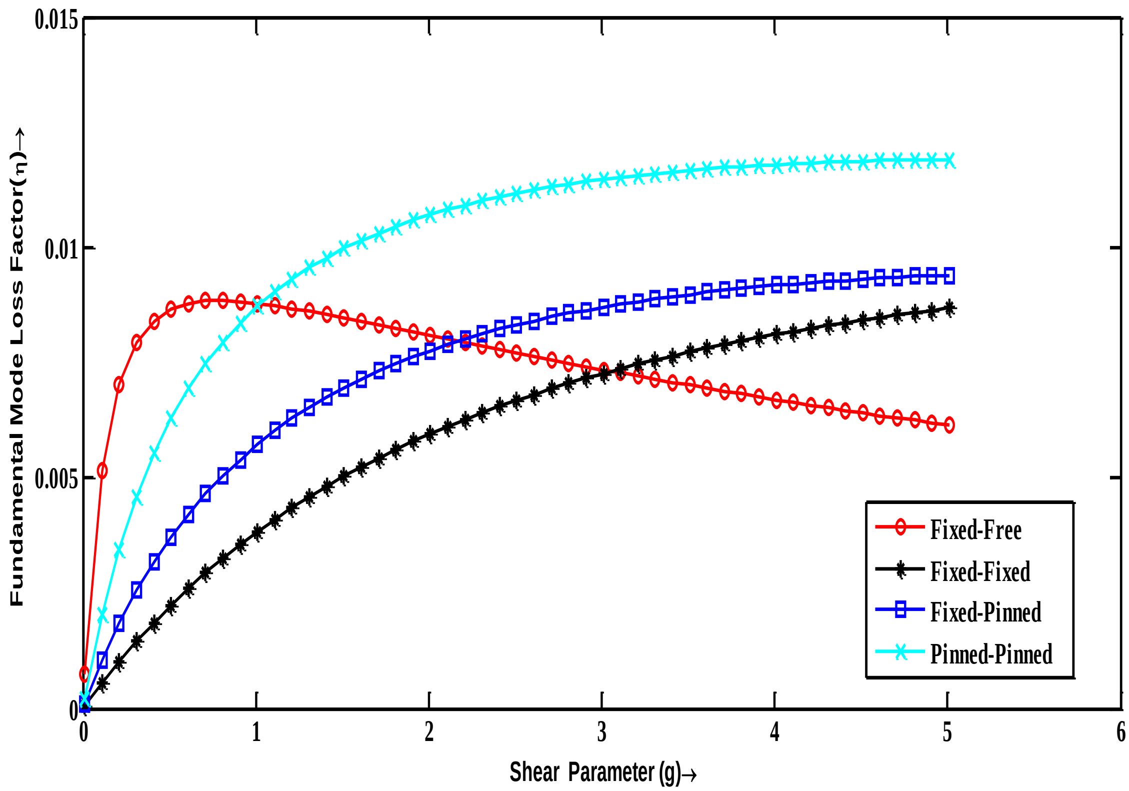

Figure 13 Shear parameter Vs fundamental loss factor, t2/t1=1

Chapter 5 & 6

Conclusion & Future Scope

5.0 Conclusion

The present work depend upon theoretical investigation of sandwich structure by using magneto rheological fluid as core material. The finite element method is used to analyze the model of beam. The validation of formulation is done by comparing data available in literature [36]. The theoretical investigation is done for different boundary conditions such as fixed-fixed, fixed-pinned, fixed-free and pinned-pinned.

- ✓ The fundamental buckling load parameter increases linearly w.r.t core thickness parameter.
- ✓ The fundamental mode frequency parameter increase w.r.t core thickness parameter.
- ✓ The fundamental mode loss factor increase w.r.t core thickness parameter.
- ✓ The fundamental buckling load parameter increases w.r.t magnetic field strength.
- ✓ The fundamental mode frequency parameter increase w.r.t magnetic field strength.
- ✓ The fundamental mode loss factor increase and attain maximum value then decreases w.r.t magnetic field strength except fixed free condition.
- ✓ The fundamental buckling load parameter almost increases linearly w.r.t shear parameter.
- ✓ The fundamental mode frequency parameter increase w.r.t shear parameter.
- ✓ The fundamental mode loss factor increase w.r.t. shear parameter except fixed free condition.

6.0 Future scope:

- The beam can be analyzed by forced vibration.
- The analysis can be extended to plate and shell.
- Experimental investigation of dynamic and buckling on different MR fluid.

Chapter No. 7

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